Uncertainty Quantification and Experimental Design for Large-Scale Linear Inverse Problems under Gaussian Process Priors, with Applications to Geophysics.

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Joint work with D. Ginsbourger (UniBe) and N. Linde (UniL)

Goal: Find high-density regions in Stromboli

Where should we collect data?

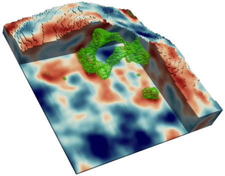


Figure: True density and excursion set (generated from model).

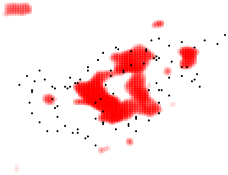


Figure: Proposed data collection plan, wIVR long-range strategy, total budget of 90 observations.

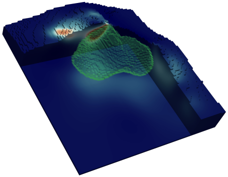


Figure: Estimated excursion set (Vorob'ev Expectation) and coverage function.

- Extend Bayesian experimental design criterion to inverse problems.
- Fine grained reconstruction (50m) $\implies$ big grid ($\sim$ 150k cells) $\implies$ big cov matrix ($\sim$ 150GB).

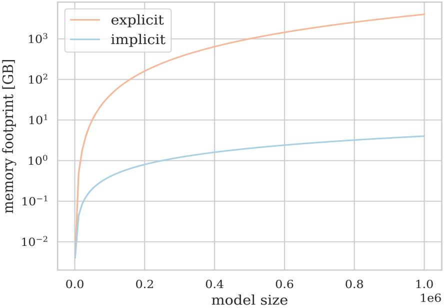
Implicit Representation of Covariance Matrix

Observation

Posterior covariance information may be extracted via products with *tall and thin* matrices:

$$\tilde{K}A, \ A \in \mathbb{R}^{m \times p}, \ p \ll m$$

Only need to maintain a multiplication routine.



Implicit Representation: Sequential Setting

$$K^{(n)}A = K^{(0)}A - \sum_{i=1}^{n} \bar{K}_i R_i^{-1} \bar{K}_i^T A$$

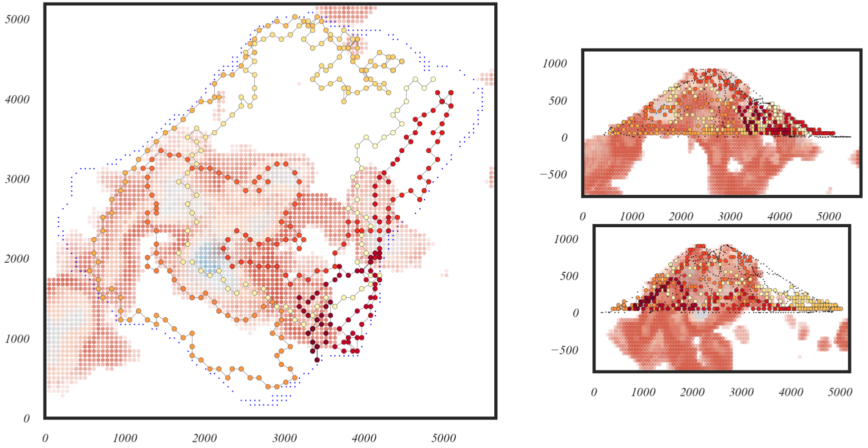
$$\bar{K}_i := K^{(i-1)} G_i^T,$$

$$R_i^{-1} := \left(G_i K^{(i-1)} G_i^T + \tau^2 I\right)^{-1}.$$

- Update representation at evey new data inclusion.

Bayesian Experimental Design for Inverse Problems

Implicit Representation gives access to (and fast updates of) posterior covariance on large grids.



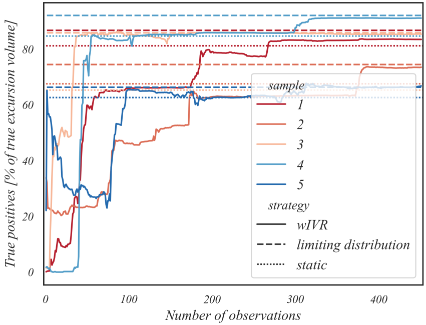
Diagnostics: Limiting Distribution

Since implicit representation allows fast inclusion of new datapoints, can study the *limiting distribution*.

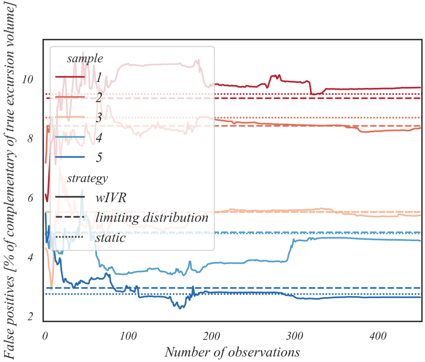
Limiting Distribution

- Posterior distribution after data collected at all accessible locations.
- Gives sense of *minimal residual uncertainty* (inherent to this type of data).

Results



(a) True positives

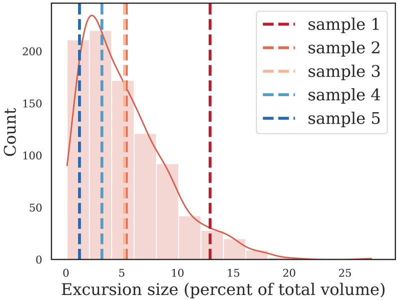


(b) False positives

Figure: Evolution of true and false positives as a function of the number of observations.

UQ on Excursion Volume

Implicit representation allows sampling through residual kriging.



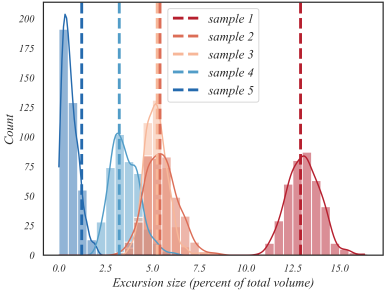


Figure: Prior (left) and empirical posterior (right) distribution (after 450 observations) of the excursion volume for each ground truth. True volumes are denoted by vertical lines.